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DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION Unclassified		1b. RESTRICTIVE MARKINGS	
2a. SECURITY CLASSIFICATION AUTHORITY JAN 14 1992		3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.	
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE		5. MONITORING ORGANIZATION REPORT NUMBER(S) ARO 25203.17-MA	
4. PERFORMING ORGANIZATION REPORT NUMBER NAMS-7 /T19		7a. NAME OF MONITORING ORGANIZATION U. S. Army Research Office	
6a. NAME OF PERFORMING ORGANIZATION Carnegie Mellon University		7b. ADDRESS (City, State, and ZIP Code) P. O. Box 12211 Research Triangle Park, NC 27709-2211	
6b. OFFICE SYMBOL (If applicable)		9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER DAAL03-88-K-0048	
8a. NAME OF FUNDING/SPONSORING ORGANIZATION U. S. Army Research Office		10. SOURCE OF FUNDING NUMBERS	
8b. OFFICE SYMBOL (If applicable)		PROGRAM ELEMENT NO.	PROJECT NO.
8c. ADDRESS (City, State, and ZIP Code) P. O. Box 12211 Research Triangle Park, NC 27709-2211		TASK NO.	WORK UNIT ACCESSION NO.
11. TITLE (Include Security Classification) Some Remarks on the Stefan Problem with Surface Structure			
12. PERSONAL AUTHOR(S) Morton E. Gurtin and H. Mete Soner			
13a. TYPE OF REPORT Final	13b. TIME COVERED FROM 15 Apr 88 TO 14 Apr 91	14. DATE OF REPORT (Year, Month, Day) November 1990	15. PAGE COUNT 24
16. SUPPLEMENTARY NOTATION The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.			
17. COSATI CODES		18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	SUB-GROUP	
		Phase transitions, Stefan Problem, surface energy, supercooling	
19. ABSTRACT (Continue on reverse if necessary and identify by block number) This paper discusses a generalized Stefan problem which allows for supercooling and superheating and for capillarity in the interface between phases. Simple solutions are obtained indicating the chief differences between this problem and the classical Stefan problem. A weak formulation of the general problem is given.			
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT <input type="checkbox"/> UNCLASSIFIED/DUNLIMITED <input type="checkbox"/> SAME AS RPT. <input type="checkbox"/> DTIC USERS		21. ABSTRACT SECURITY CLASSIFICATION Unclassified	
22a. NAME OF RESPONSIBLE INDIVIDUAL		22b. TELEPHONE (Include Area Code)	22c. OFFICE SYMBOL

FINAL PROGRESS REPORT

1. **ARO PROPOSAL NUMBER:** 25203-MA
2. **TITLE OF PROPOSAL** Stability and Thermal Influences in
Nonlinear Continuum Mechanics
3. **CONTRACT OR GRANT NUMBER:** DAAL03-88-K-0048
4. **NAME OF INSTITUTION:** Carnegie Mellon University
5. **AUTHORS OF REPORT** Morton E. Gurtin
6. **LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO
SPONSORSHIP DURING THIS REPORTING PERIOD, INCLUDING JOURNAL
REFERENCES:**

See attached page.



7. **SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES
AWARDED DURING THIS REPORTING PERIOD:**

Allan Struthers, Ph.D. in Mathematics, May 19, 1991
Jose Matias, currently pursuing a Ph.D.

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Brief Outline of Research Findings

Work continued on the thermodynamics of two-phase continua. The general two-phase Stefan problem with supercooling, superheating, and capillarity, was studied [T19] in collaboration with M. Soner (Carnegie Mellon). Simple solutions — illustrating the chief differences between this problem and the classical Stefan problem — were obtained for the spherically symmetric problem, under the assumption of fast diffusion, with the liquid supercooled at infinity. It is shown that: (i) for $\Omega = \mathbb{R}^3$, a ball of the solid phase of sufficiently small size disappears in finite time, but a sufficiently large ball grows without bound; (ii) for $\Omega = \mathbb{R}^3$ and the solid phase initially situated in a spherical shell of thickness ε , the thickness of the solid shell initially increases, but the inner radius of this region decreases to zero in finite time T ; the solid ball remaining at time T disappears at a later time or grows without bound according as ε is less than or greater than a critical value; in the limit $\varepsilon \rightarrow 0$ the region occupied by the solid disappears infinitely fast; the problem has no solution for $\varepsilon = 0$; (iii) when Ω is the region exterior to a sphere of radius R , with the boundary $r = R$ insulated and with the solid phase initially in a spherical shell of zero thickness at $r = R$, the solid phase grows without bound provided R is sufficiently large. While (ii) and (iii) are of little practical interest, they demonstrate the possibility of growth from a seed of zero volume.

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* Discussed in previous progress reports.